

CALCULATION POLICY

(EYFS, KS1 & KS2)

GB Committee Responsible:	Quality and Standards
Reviewed by:	Steven Pearce/Clive Westall
Review Date:	June 2020
Approved by Committee:	11th June 2020
Next Review Date of Policy:	10th June 2022

Aylward Primary School Calculation Policy

This document is a statement of the aims, principles and strategies for the teaching and learning of mathematical calculations at Aylward Primary School.

Rationale
This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the 2014 National Curriculum.

The written calculation part of the policy demonstrates the expected (Age related Expectations) methods to be used in different year groups, when recording in a book. The Progression of skills part of the policy demonstrates teaching methods and how one leads on to the next, to build up to the formal written methods. This will sometimes be visible in the children's books through drawings and photos.

School wide policies, such as this, ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding (progression of skills). However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 (Written calculation) and in line with school policy.

The importance of mental mathematics
While this policy focuses on written calculations in mathematics and the use of physical resources to support learning a topic, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The beginning of the 'Progression of skills' section outlines these key skills and the number facts that children are expected to develop throughout the school. Manipulatives are evident in the first stages of progression but will be available even when children are using written methods throughout the school.

This policy has been updated June 2020 and will be reviewed June 2022.

Staff responsible
Mr. C Westall - Head Teacher
Mr. S Pearce - Maths Leader
Mr. M Borgese & Mr. D Bendelman - Governors

Content

Written calculation

Addition: Pages 4 -6

Subtraction: Pages 7-9

Multiplication: Pages 10-14

Division: Pages 15-19

Progression of skills

Mental Maths skills: Page 20-21

Addition and Subtraction: Page 22-29

Multiplication and division: Page 30 -39

Written calculations

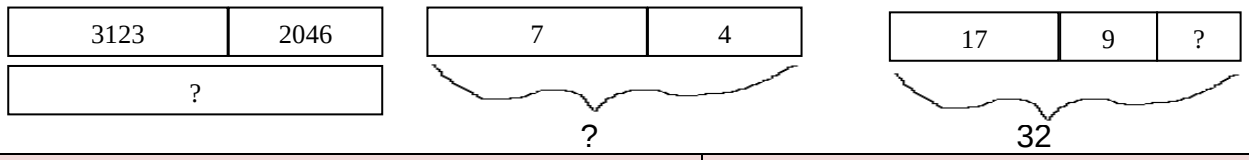
The term ones should be used not units when labelling the column.

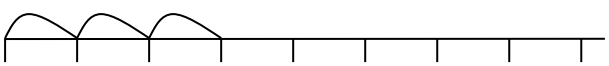
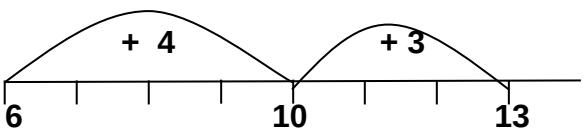
Addition

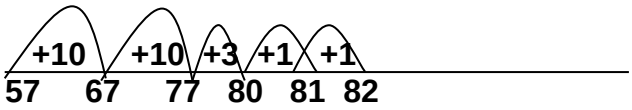
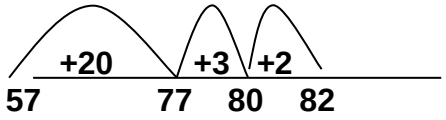
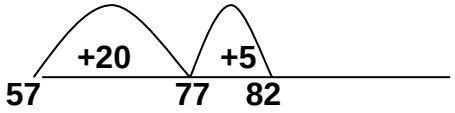
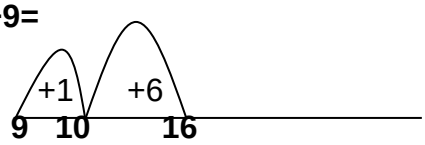
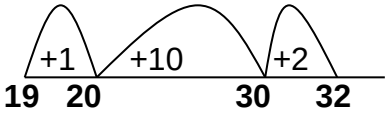
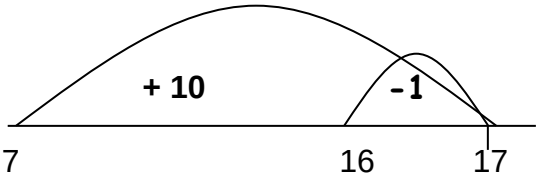
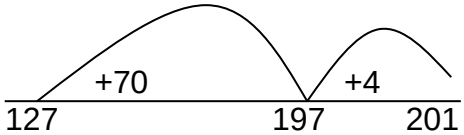
Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Bar modelling

Bar modelling is a method encouraged to be used at all ages. Its focus is on understanding the worded question, not answering it. It is used to add two or more numbers and can be presented in a variety of ways beginning with physical blocks and then moving onto drawn bars. Children add information from the text, one sentence at a time. Below are a few examples:



Calculation Method	Explanation
EYFS	
Counting Children record the total number of items next to a picture.  = 4	Showing an understanding of the five principles (see progression of skills policy). Children will develop on from counting real objects to counting objects in pictures.
4 + 1 = Counting one more: 	Children to write the sum but to use tangible objects to calculate one more.
2 + 3 = At a party, I eat 2 cakes and my friend eats 3. How many cakes did we eat altogether? 	Children draw a picture to help them work out the answer.
Year 1	
7 + 4 = 7 people are on the bus. 4 more get on at the next stop. How many people are on the bus now? ……. …. We would then progress to; **7 +…. = 11**	Children use dots or symbols to represent objects (quicker than drawing a picture) We would then progress to counting on from 7.
5 + 3 = +1 +1 +1  **5 6 7 8 9 10 11** **6 + 4 + 3 =** 	Children use a number line to record jumps made. Using single jumps… or larger jumps.

Year 1/2	
57 + 25 = My sunflower is 57cm tall. It grows another 25cm. How tall is it now? **Method 1**  **Method 2** **57 +25 =**  **Method 3** **57 + 25 =**  	Drawing an **empty number line** helps children to record the steps they have taken in a calculation. Start on 57 count on a jump of 10 to land on 67. Then add on another jump of 10 to land on 77. Then add on 3 to land on 80 then a jump of 1 to land on 81 and 1 to land on 82. Start on 57. Add on 20 to land on 77. Then add 3 to get to 80 then 2. This is much more efficient than counting on in ones. At all times it is important to reinforce number bonds to 10. Start on 57. Add on 20 to land on 77 then add 5 to land on 82. Children may also **partition** numbers – **57 + 25 =** **50 + 20 = 70** **7 + 5 = 12** **70 + 12 = 82**
7+9=  **13 + 19=**  	Draw an empty number line. Children will be taught to place the largest number at the left side of the line. Add 1 to get to 10. Then add the 6. This method is also used when adding 19, 29, 39 etc
7 + 9 = (7 + 10) – 1= 16  (Using number line to record)	Children use a sound understanding of adding 10 to adapt calculations. Instead of adding 9, they add 10 then **adjust** the calculation by subtracting 1. This is also used for adding other numbers e.g. 15 + 19 (15 **+ 20**, - 1) This is a mental strategy
127 + 74 = There are 127 boys and 74 girls in a school. How many children are there altogether?  	Children will start their number line at 127. Add a jump of 70 to 197. Either add 3 then 1 or simply a jump of 4 to land on 201.

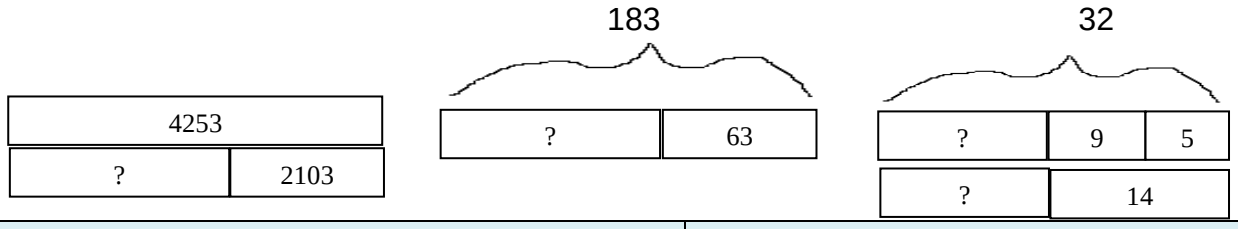
Year 2	
20 + 4 30 + 5 50 + 9 2 4 + 3 5 5 9	Children will be introduced to a formal written method. No carrying is involved at this early stage and the expanded method should be taught first to ensure children understand the place value. The value of the numbers should not exceed 2 digits.
Year 3	
1 2 7 + 7 4 100 + 20 + 7 + 70 + 4 100 + 90 + 11 =201	Children will be taught **written method**s for those calculations they cannot do 'in their heads'. **Expanded methods** build on mental methods and make the value of the digits clear to children. The language used is very important **(7+4, 20+70, 100+0, then 100+90+11 - *add this mentally*** *** In this expanded method, when children get to the stage of adding their partitioned digits together they may be required to 'carry' numbers (see below).** The value of the numbers should not exceed 3 digits.
2 9 8 3 5 8 6 5 6 1 1	When children are confident using the expanded method, this can be 'squashed' into the traditional compact method.
Year 4	
2786 + 2568 = 2786 people visited the museum last month. The numbers increased by 2568 this month. How many people altogether visited this month? 2 7 8 6 + 2 5 6 8 5 3 5 4 1 1 1	When children are confident using the expanded method, this can be 'squashed' into the traditional compact method. (Carrying!) Columns should be referred to. It is not 7+5, it is 7 lots of hundred + 5 lots of hundred The value of the numbers should not exceed 4 digits.
Year 5	
2 0 + 4 + 0 . 5 3 0 + 9 + 0 . 8 5 0 + 1 3 + 1 . 3 = 6 4 . 3 2 4 . 5 3 9 . 8 6 4 . 3 1 1	Add whole numbers up to 5 digits. Decimals numbers will be introduced this year (money problems will include them at younger ages). Children will start with the expanded method to ensure their understanding of place value is secure before moving onto the compact method.
Year 6	
2 4 . 5 6 6 3 9 . 7 0 0 0 . 5 6 0 6 4 . 8 2 6 1 1 1	Year 6 will use the compact method and add larger numbers and decimals up to 3 places. In order to keep the place value, the children may add 0s in the empty decimal columns.

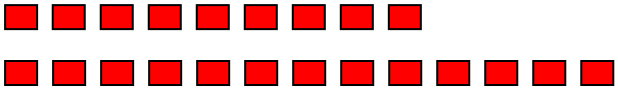
Subtraction

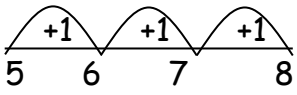
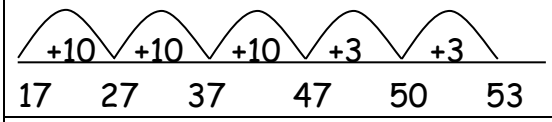
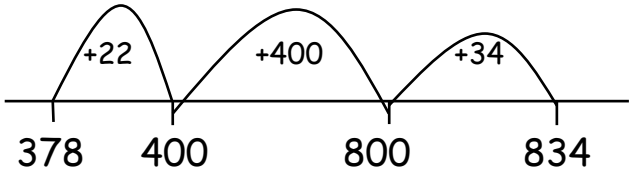
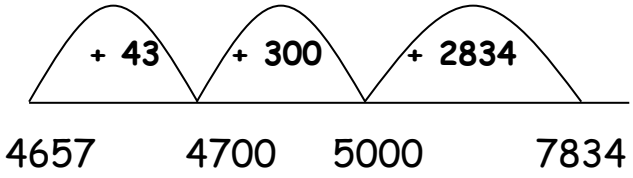
Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

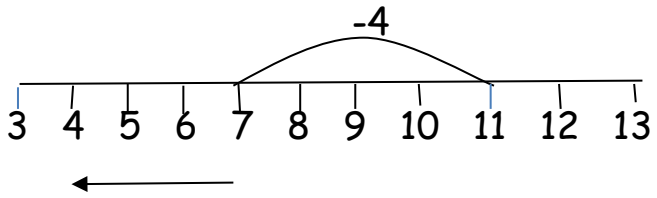
Bar modelling

Bar modelling is a method encouraged to be used at all ages. Its focus is on understanding the worded question, not answering it. It is used to record the starting number at the top. The number/s subtracted and the answer are recorded below. Here are a few examples:



Calculation Method	Explanation
EYFS	
4 - 1 = Counting one less:  removed	Children to write the sum but to use tangible objects to calculate one less.
5 - 2 = I had five balloons. Two burst. How many did I have left?  Take away	Drawing a picture helps children to visualise the problem. Children work on counting back to also take away.
Year 1	
A teddy bear costs £5 and a doll costs £3. How much more does the bear cost?  ? ? Find the difference	Drawing a picture helps children to visualise the problem.
7 - 3 = Mum baked 7 biscuits. I ate 3. How many were left?  Take away Lisa has 7 felt tip pens and Tim has 3. How many more does Lisa have? ●●●●●●● ●●● Find the difference Sarah has 9p and John has 13p. How much more does John have? 	Using dots or tally marks is quicker than drawing a detailed picture.

Year 1/2	
COUNTING UP/Find the difference **8-5= 3**  **53-17= 36**  	Children are taught to **count up** in order to subtract or '**find the difference**'. The children count up from 5 in jumps of 1 to get to 8. They then count the jumps to get the answer. Count up from 17 by adding 3 jumps of 10 (or simply a jump of 30). Then, 2 jumps of 3 to reach 53. Add the jumps up to get 36 which is the difference between the two numbers.
834 - 378 = The library owns 834 books. 378 are out on loan. How many are on the shelves?  **7834 – 4657=**  	As the children progress through the school, the counting up method could be used for longer numbers when appropriate. Children could **count up** (from the smallest number to the biggest) using an empty number line. It is easiest to count up to a multiple of 10 or 100 (a friendly number). This method can be used with any numbers, even decimals.
Year 2	
80 + 9 **- 50 + 5** **30 + 4 = 34** **8 9** **- 5 5** **3 4**	Children will be introduced to a formal written method. No 'borrowing' (decomposition) from other columns is introduced at this early stage and the expanded method should be taught first to ensure children understand the place value. The value of the numbers should not exceed 2 digits.
Year 3	
Expanded decomposition **749** **- 273** ——— 600 140 ~~700~~ + ~~40~~ + 9 200 + 70 + 3 400 + 70+ 6 = 476	When children are secure using the column method, expanded decomposition is introduced. Partition the numbers and set out in columns. Always start subtracting the ones first. If the top number is smaller than the bottom number, then you will need to take from the

	column on the left. Remember to adjust the columns to show the new amounts. Finally, recombine the answers to give your final answer. The value of the numbers should not exceed 3 digits.
Decomposition 6 1 5 ~~7~~ 2 - 2 4 5 3 2 7	Once the children are secure with expanded, they can move on to the shortened/compact method. This is just the same as above but the numbers are not partitioned out to see the different columns as clearly.
Year 4	
2 1 3 1 ~~3~~ 5 ~~4~~ 7 - 2 6 2 8 0 9 1 9	The children will use the same methods as year 3; column and decomposition but the value of the digits should not exceed 4 digits.
Year 5	
2 1 3 1 7 1 ~~3~~ 5 ~~4~~ 7.8 ~~8~~ 5 - 2 6 2 8.2 7 8 0 9 1 9.6 0 7	The children will use the same methods as in years 3 and 4; column and decomposition but the value of the digits should not exceed 5 digits. Decimals will be introduced up to 3 decimal places – in this instance, expanded decomposition should be taught first to ensure all children understand the place value.
Year 6	
2 1 3 1 7 1 ~~3~~ 5 ~~4~~ 7.8 ~~8~~ 5 - 2 6 2 8.2 7 8 0 9 1 9.6 0 7 7 - -4  	The children will use the same methods as in years 3, 4 and 5; column and decomposition with a range of larger numbers. They will also add and subtract negative integers – this will be done using a number line to begin with.

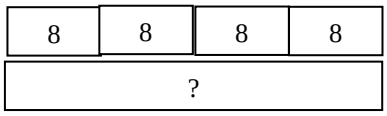
Multiplication

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

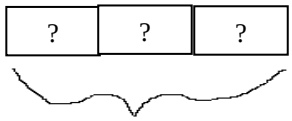
Bar modelling

Bar modelling is a method encouraged to be used at all ages. Its focus is on understanding the worded question, not answering it. Here are a few examples showing how it can be used to show a worded multiplication problem:

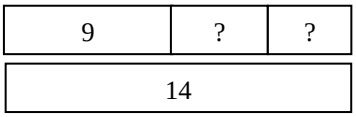
8 x 4 = ?

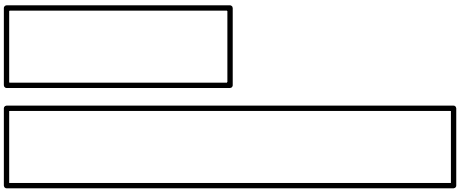
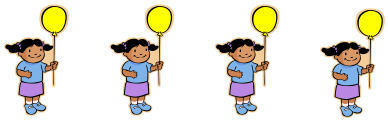
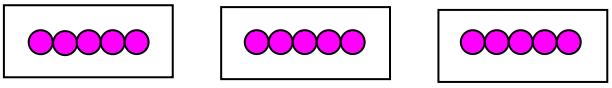


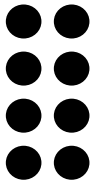
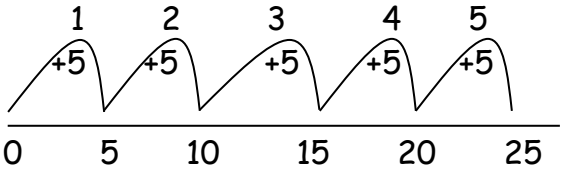
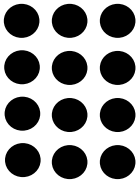
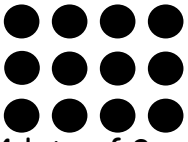
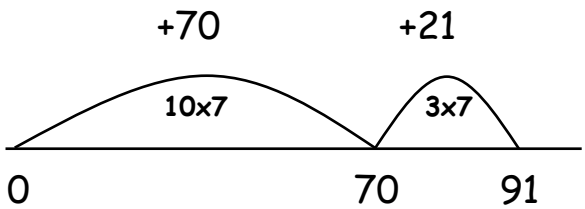
3 x __ = 63



9 + 2M = 14



Calculation Method	Explanation
EYFS	
Doubling 	Children draw or stick in something before repeating with double the amount.
Year 1	
2 x 4 = Each child has two feet. How many feet do four children have?  **2 + 2 + 2 + 2**	A **picture** can also be useful for early multiplication. We say "2 four times."
5 x 3 = There are 5 cakes in a pack. How many cakes in 3 packs?  **5 + 5 + 5**	Dots or tally marks are often drawn in groups. This shows 3 groups of 5.

2 x 4 = A chew costs 4p. How much do 2 chews cost?  2 lots of 4 2 sets of 4	Drawing an **array** (**4 rows of 2** or **2 columns of 4**) give children an **image** of the answer. It also helps **develop the understanding that 4x2 is equivalent to 2x4.**
5 times table visual aid. 	The children will be taught their times tables and division facts using a number line as a visual aid.
Year 2	
3 x 4 = A chew costs 4p. How much do 3 chews cost?  3 lots of 4 3 sets of 4  4 lots of 3 4 sets of 3	Drawing an **array** (**4 rows of 3** or **3 columns of 4**) gives children an **image** of the answer. It also helps **develop the understanding that 4x3 is equivalent to 3x4.**
13 x 7 = 91 There are 13 biscuits in a packet. How many biscuits in 7 packets? 	When numbers get bigger, it is inefficient to do lots of small jumps. **Split 13 into parts** (<u>10 and 3</u>). This gives you **two jumps** (**10x7** and **3x7**). The answer is the number you land on 91.

Year 3	
Mental Strategy 26 x 7 = Partitioning. 20 x 7= 140 6x 7= 42 140 + 42= 182 KNOWN FACTS 2 x 7 = 14 20 x 7=140	Children use the partitioning method to multiply mentally and when multiplying by a ones (single digit). Partition 26 (into 20 and 6) and use KNOWN FACTS to multiply by 7. 2x7=14 so 20 x 7=140 6x7= 42 Then add the answers together 140 + 42 = 182. This is a mental strategy
Mental Strategy Grid multiplication 54 x 7 = KNOWN FACTS 5 x 7 = 35 5 x 70 = 350 X | 50 | 4 7 | 350 | 28 = 378	Split up the numbers and multiply each part. Remember to write down KNOWN FACTS. **Add across the rows, then add those two answers together.**
Short Multiplication H T U 3 4 x 6 2 0 4 2 2	Children use short multiplication to multiply TO x O Starting with the ones column Ones x Ones (6 x 4= 24). Place the 4 from 24 into the answer in the ones column and carry the 2 tens below the answer in the tens column. Then multiply T x O (30 x 6 = 180). Add your 2 carried tens from below the answer column to equal 200. This is 20 Tens therefore you write 20 in the answer column under H and T.
Year 4	
Short Multiplication H T U 3 4 x 6 2 0 4 2 2	Children use short multiplication to multiply TU x U **See above** *When teaching the children our dialogue would be:* *Step 1: O x O 4 x 6 = 24, put the 4 in the answer section for ones and carry the 20 to become 2 tens.* *Step 2: T X U 3 x 6= 18 lots of ten, add the 2 lots of tens = 20 lots of ten. Put the 20 in the answer box or show how this is 0 tens and 2 lots of a hundred. Final answer= 204.*

Short Multiplication Th H T U 3 2 5 X 7 2 2 7 5 1 3	Children use short multiplication to multiply HTO x O To understand place value we would explain the following: Starting with O x O (5 x 7= 35). Place the 5 from the 35 into the answer in the ones column and carry the 3 tens **<u>below</u>** the answer in the tens column. Then multiply T x O (20 x 7= 140) Add your carried 3 tens (140 +30=170) Place the 7 Tens in the Tens column of your answer and carry the 1 hundred below the answer in the hundred's column. Next multiply the H x O (300 x 7= 2100). This is 21 hundreds, so now add the carried 1 hundred to make 22 hundreds (2200). Finally place the 22 under the Th and H columns. Final answer = 2275 *When teaching the children our dialogue would be:* *Step 1- O x O 5 x 7 = 35 put 5 in the answer carry the 3.* *Step 2 T x O 2 X 7 = 14 lots of ten add the 3 = 17 lots of ten. Put the 7 in the tens column and carry the 1 lot of hundred.* *Step 3- H x O 3 x 7=21 lots of hundred add the 1 = 22 lots of hundred. Write 22 in the answer box* *Final answer = 2275*
Year 5	
Short Multiplication Th H T U 3 2 5 X 7 2 2 7 5 2 1 3	Children use short multiplication to multiply HTU x U **<u>See above</u>**
Long Multiplication. 59 x 26 Th H T U 5 9 X 2 6 3 5 3 5 4 1 1 1 1 8 0 1 5 3 4 1 1	*When teaching the children our dialogue would be:* *Step 1: O x O 6 x 9= 54 put the 4 in the answer and carry the 5* ***above the line.*** *Step 2: O x T 6 x 5= 30. Add the 5 to make 35 lots of ten. Put 35 in the answer, First line answer 354.* *Step 3: Put a 0 in the answer under the ones column (or the first column if there are decimals) to hold the place value as you are about to multiply everything by 10. The children should have a strong understanding why before doing it in writing.* *Step 4: T x O 2 x 9=18, put 8 in the tens, carry the 10 tens to become 1 lot of a hundred above the line.* *Step 5: T X T 2 X 5= 10, add the 1 = 11 lots of a hundred. Put 11 in the answer. Second line answer 1180.* *Step 6: add the first and second line answers together. Final answer= 1534.*

Year 6 As year 5 with larger numbers and decimals	

Although Long and short multiplication are the focus of the curriculum due to them being quicker, the grid method still is important. It is also very effective when they come to algebra.

When using column methods is vital that children know the columns they are working with and it is 7 lots of ten not just the digit 7 for example.

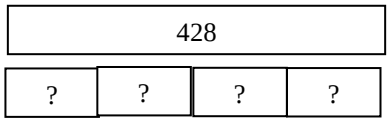
Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.
The use of manipulatives is so important for division and children should always have access to these. They should be using them when looking at counting up or down on a number line.

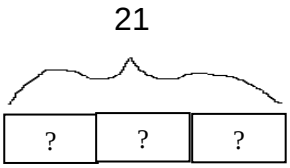
Bar modelling

Bar modelling is a method encouraged to be used at all ages. Its focus is on understanding the worded question, not answering it. Children work a sentence at a time to record information until they understand what is being asked and use a taught method to answer it. Here are a few examples showing how it can be set out for division:

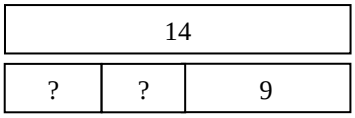
428 ÷ 4 = ?

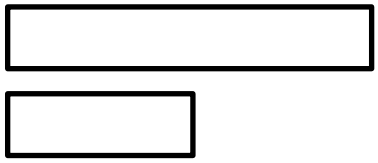


21 ÷ 3 = ?



(14 – 9) ÷2 = ?



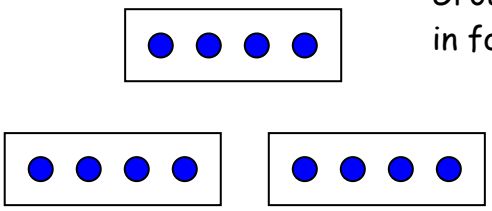
Calculation Method	Explanation
EYFS	
Halving  	Children start by finding half of real items. They will often cut the items to show it and pictures will be used. After sticking in paper, cut another sheet of paper to show half. Some will complete the task with just one sheet.
Year 1	
6 ÷ 2 = 6 Easter eggs are shared between 2 children. How many eggs do they get each?  Sharing between 2 There are 6 Easter eggs. How many children can have two each?  Grouping in twos	More pictures! Drawing often gives children a way into solving the problem. Children are also able to draw circles to show the two groups and then draw out the 6 eggs (or dots for speed) into them. Children would take two eggs then see if they can take another two to make another group. When they can no longer take eggs, they count how many groups they have.

Year 1 and 2

12 ÷ 4 =

4 apples are packed in a basket.
How many baskets can you fill with 12 apples?

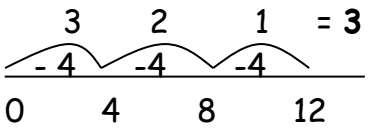
Grouping in fours



Dots or tally marks can either be shared out one at a time or split up into groups.

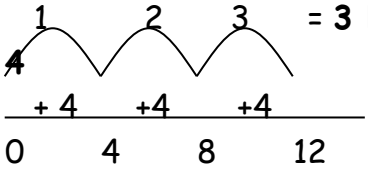
Year 2

12 ÷ 4= 3



Using the pictorial example above, we would then progress to transferring this to a number line by taking out sets of 4 and counting down.

12 ÷ 4= 3



The above method would progress into a number line. When you add on **'lots of 4' or 'sets of 4'** until you reach 12. You then count up how many lots of 4 you have added on to get the answer **3**.

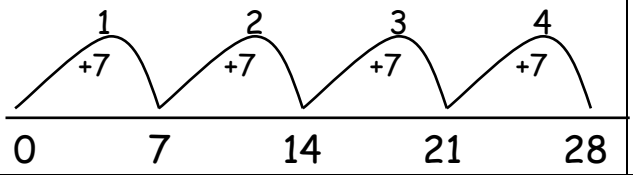
Many children also still use the grouping method with circles, shown in Year 1.

Year 3

Mental Strategy

28 ÷ 7 = 4

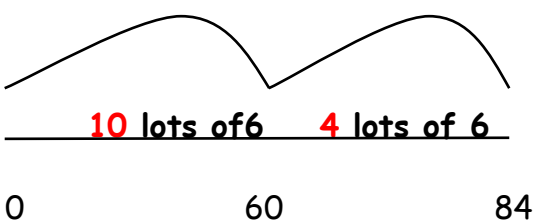
A chew bar costs 7p. How many can I buy with 28p?



To work out how many lots of 7 there are in 28, draw **jumps** of 7 along a **number line. This shows you need 4 jumps of 7 to reach 28.**

Mental Strategy

84 ÷ 6 =



It would take a long time for the children to jump in sixes to 84 so children can jump on in bigger '**jumps**'.

A jump of 10 groups of 6 takes you to 60. Then you need another 4 lots of 6 which is 24 will take you to 84. Altogether this is **14 sixes.**

Calculations should be 2 digit by 1 digit.

Year 3/4

Short Division

$$\begin{array}{r} 19 \\ 4\overline{)\,7^{3}6} \end{array}$$

Teacher would explain to the children that we ignore place value when teaching short division as a strategy once the children have fully understood place value.

Firstly, work out how many fours fit into 7. Write the **answer above** 7 and the **reminder** in front of the next digit of the number to be divided.

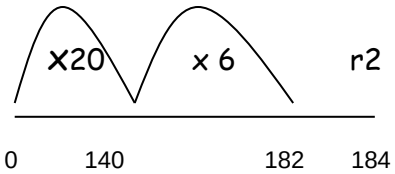
Then count up in fours to see how many now fit into 36 and write the answer above the digit 6. They should be encouraged to use their 4 times table knowledge.
If the divisor does not fit exactly into the final number, you will be left with a remainder to be recorded next to your answer.

Year 4/5

Mental Strategy

184 ÷ 7 =

I need 184 chairs for a concert. I arrange them in rows of 7. How many rows do I need?



KNOWN FACTS
7 x 2 = 14
7 x 20=140

If the number you are dividing by (7) cannot go into the number any more, this is the remainder!

This method is known as **counting up on a number line.**

In this example, you are counting in jumps of 7.

Using their **KNOWN FACTS** children will know 7 x 2=14 so 7 x 20=140

So first add on a jump of 20 lots of 7 and land on 140.

You are left with 44. Cont...

Then add on a jump of 6 lots of 7 which is 42 and land on 182, to leave 2.

Altogether, that is 26 sevens with a remainder of 2.

Calculations should be 2 or 3 digit by 1 digit and children should be taught to interpret the remainder appropriately.

In year 5 children will divide up to 4 digits by a 1 digit number and 10 and interpret remainders appropriately.

Year 5/6

Short Division

$$\begin{array}{r} 19 \\ 4\overline{)\,7^{3}6} \end{array}$$

First, work out how many fours fit into 7. Write the **answer above** 7 and the **reminder** in front of the next digit of the number to be divided.

Teacher would explain to the children that we ignore place value when teaching short division as a strategy once the children have fully understood place value.

Then count up in fours to see how many now fit into 36 and write the answer above the digit 6.

They should be encouraged to use their 4 times table knowledge.
If the divisor does not fit exactly into the final number, you will be left with a remainder to be recorded next to your answer.
Remainders can be interpreted as fractions, decimals or rounding.

E.g. 36 ÷ 5 = 7 r1
= 7 1/5
= 7.2
= 7 (rounded to the nearest whole number)

$$\begin{array}{r} 120\text{ r }5 \\ 8\overline{)\,9^{1}65} \end{array}$$

Remainders can be interpreted as fractions, decimals or rounding.

E.g. 965 ÷ 8 = 120 r5
= 120 5/8
= 120.625
= 121 (rounded to the nearest whole number)

Examples will be taught and practised that include 0 and when divisors divide exactly.

Year 5/6	
Long Division 0 3 1 r 3 15 | 4 6 8 4 5 0 (30 lots of 15) 0 1 8 1 5 (1 lot of 15) 0 3	Teacher would explain to the children that we ignore place value when teaching long division as a strategy once the children have fully understood place value. Long division requires the children to be competent and confident with their tables, and subtraction before they can use it as a division strategy. 468 ÷ 15 How many 15s in 4? The answer is 0 so this is placed above the 4 (above the division gate). The next question is how many 15s are in 46? (We have put the 4 and 6 together to make 46 lots of ten). The answer is 3 lots of ten, so this is placed above the 6 (above the division gate). 30 x 15 is 450, this is written under the 468 and a subtraction calculation is done to work out the remainder which is 10. The remaining digit (8) is then brought down to join the 10 to form 18. So the next question we ask is; how many 15s are there in 18? The answer is 1, so this is written above the 8 (above the division gate). 1 x 15 is 15, this is written under the 18 and a subtraction calculation is done to work out the final remainder, which in this example is 3. This remainder could then be represented by 3/15 or then simplified to 1/5 or then put into a decimal of 31.2

Progression of Skills

To add and subtract successfully, children should be able to do the following Mentally.

Recall all addition pairs to 9 + 9…



…and number bonds up to 10:

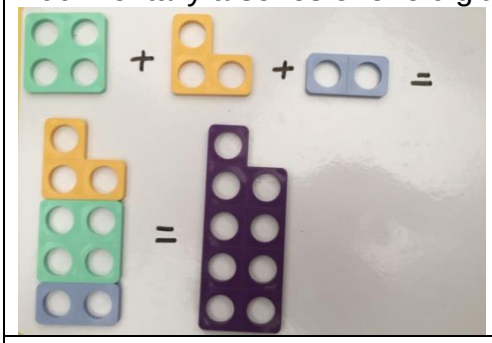


Recognise addition and subtraction as inverse operations:

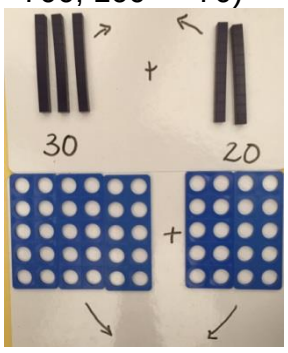
5 + 4 = 9 4 + 5 = 9

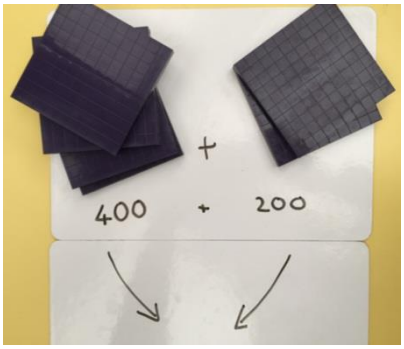
9 - 5 = 4 9 - 4 = 5

Add mentally a series of one digit numbers (e.g. 4 + 3 + 2) : Example of a game here

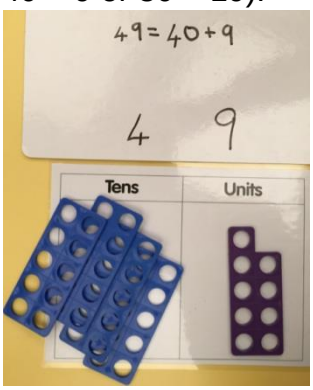


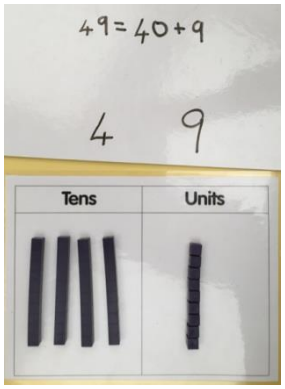
Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)

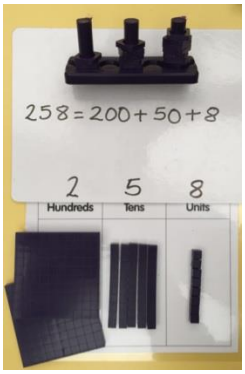




Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 49 into 40 + 9 or 30 + 19):



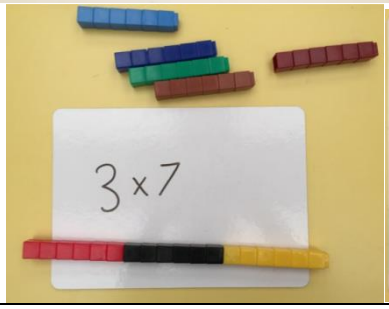
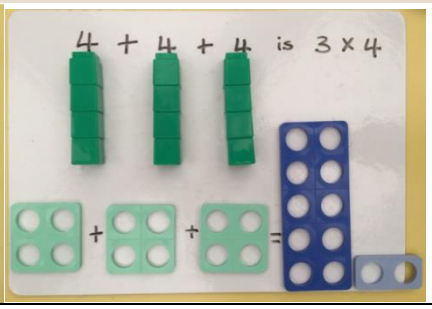
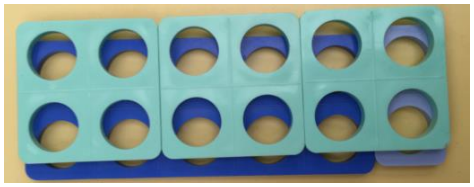
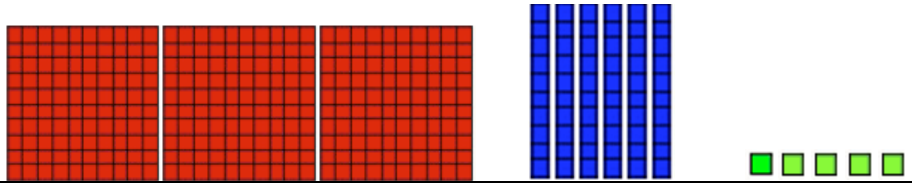




Use estimation by rounding to check answers are reasonable:

93 + 21 is about 90 + 20 so should be around 110

To multiply and divide successfully, children should be able to do the following mentally.

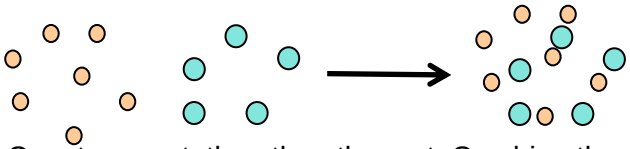
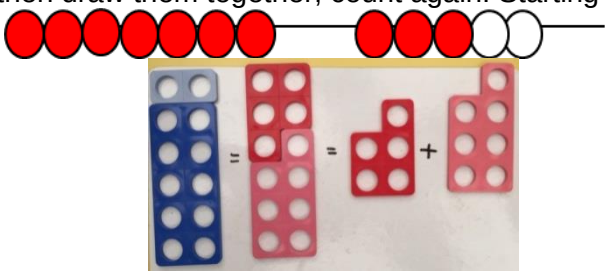
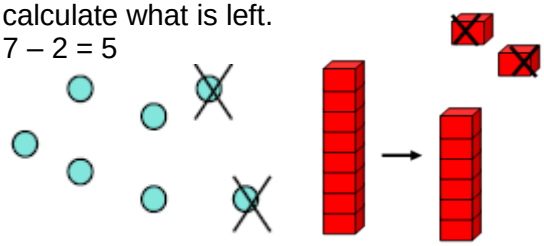
Add and subtract accurately and efficiently.
Recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12   
Use multiplication and division facts to estimate how many times one number divides into another etc. 
Know the outcome of multiplying by 0 and by 1 and of dividing by 1
Understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
Recognise factor pairs of numbers (e.g. 40 = 10 x 4 or 20 x 2 or 5 x 8 etc.) and increasingly able to recognise common factors (e.g. The common factors of 15 and 40 are 1 and 5)
Derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000) E.g. if 4 x 6 is 24 then 0.4 x 6 is 2.4 or 40 x 6 is 240
Notice and recall with increasing fluency inverse facts
Partition numbers into 100s, 10s and 1s or multiple groupings 365 = 300 + 60 + 5 
Understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division – See later in the document
Understand the effects of scaling by whole numbers and decimal numbers or fractions– See later in the document
Understand correspondence where n objects are related to m objects If  = then  = ?
Investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.

Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

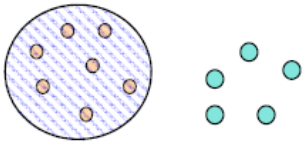
Addition	Subtraction
The five principles that govern and define counting • The One–One Principle: recognising an object as one and only one. • The Stable-Order Principle: counting 1, 2, 3 etc instead of 1, 3, 2 • The Cardinal Principle: number of final object represents the total number of objects. • The Abstraction Principle: Can be applied to any collection of objects whether tangible or not. • The Order-Irrelevance Principle: The order in which the items are counted is irrelevant. *Gelman, R. & Gallistel, C. (1978): The Child's Understanding of Number*	
1 more After counting a group of items the child can add one more item and say how many there are now.  6  add 1 more	**1 less** After counting a group of items the child can remove one item and say how many there are.  Remove one toy and count back instead of recounting
Combining two sets (aggregation) Putting together – two or more amounts or numbers are put together to make a total 7 + 5 = 12  Count one set, then the other set. Combine the sets and count again. Starting at 1. Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1. 	**Taking away (separation model)** Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5  Multilink towers - to physically take away objects. 

Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

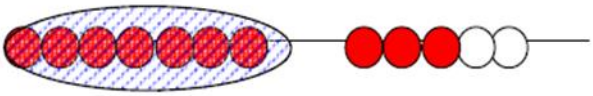
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:

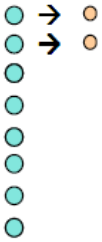


Make a set of 7 and a set of 5. Then count on from 7.

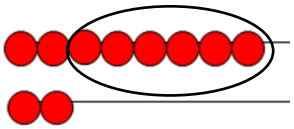
Finding the difference (comparison model)

Two quantities are compared to find the difference. 8 – 2 = 6

Counters:



Bead strings:

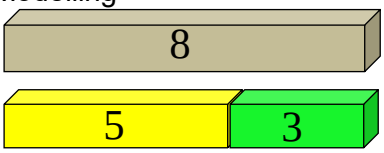


Make a set of 8 and a set of 2. Then count the gap.

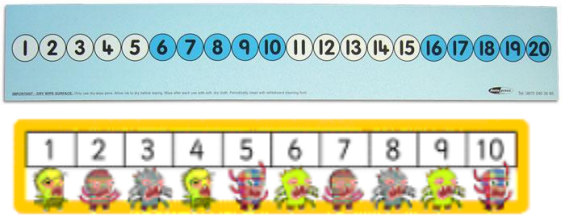
Multilink Towers:



Cuisenaire Rods: beginning links to 'Bar Modelling'

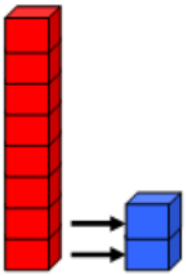


Number tracks:

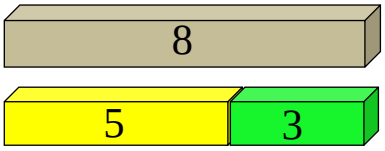


Start on 5 then count on 3 more

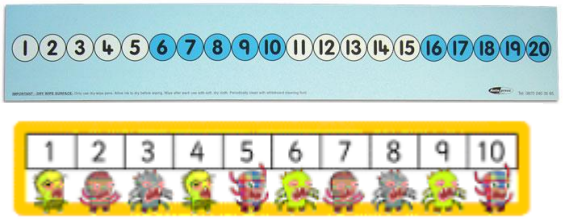
Multilink Towers:



Cuisenaire Rods: beginning links to 'Bar Modelling'



Number tracks:



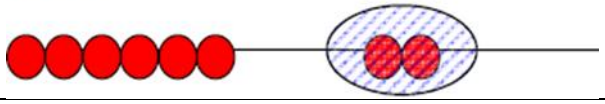
Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part. 8 – 2 = 6

Counters:

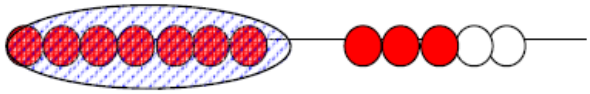


	Bead strings: 8 – 2 = 6 

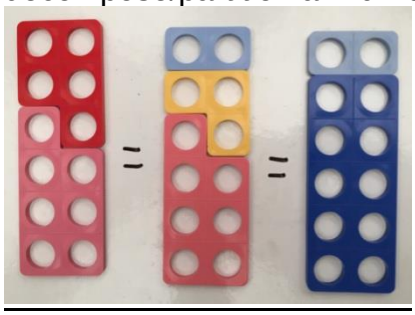
Bridging through 10s

This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



7 + 5 is decomposed / partitioned into 7 + 3 + 2.
The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

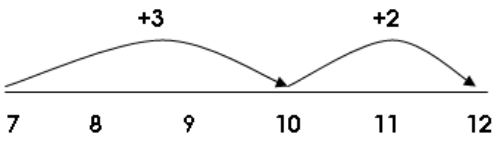


Number track:

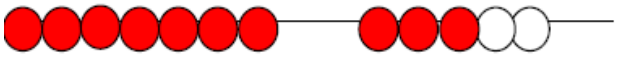


Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line

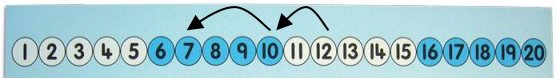


Bead string:

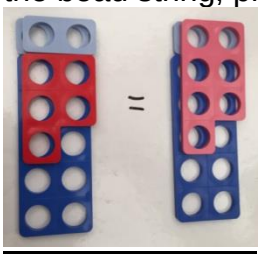


12 – 7 is decomposed / partitioned in 12 – 2 – 5.
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

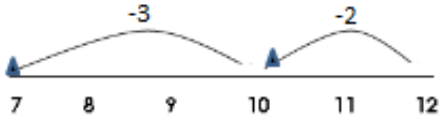
Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.



Number Line:

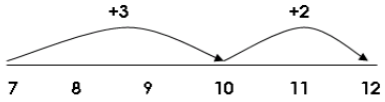


Counting up or 'Shop keepers' method

Bead string:



12 – 7 becomes 7 + 3 + 2.

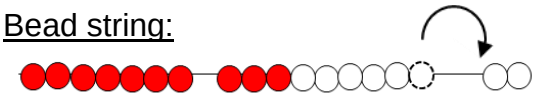
	Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'. Number Track:  Number Line: 

Compensation model (adding 9 and 11)

This model of calculation encourages efficiency and application of known facts (how to add ten)

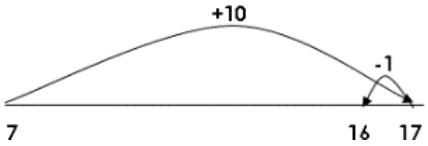
7 + 9

Bead string:



Children find 7, then add on 10 and then adjust by removing 1.

Number line:



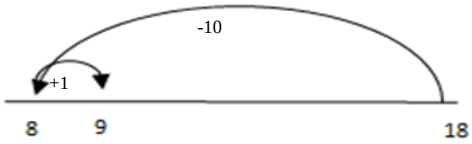
18 – 9

Bead string:



Children find 18, then subtract 10 and then adjust by adding 1.

Number line:



Working with larger numbers

Tens and ones + tens and ones

Ensure that the children have been transitioned onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

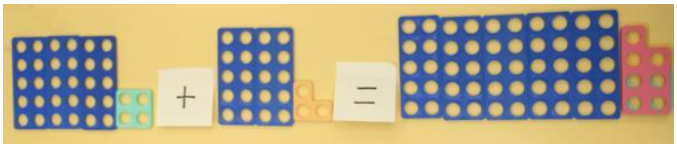
Partitioning (Aggregation model)

34 + 23 = 57

Base 10 equipment:



Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.



Take away (Separation model)

57 – 23 = 34

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

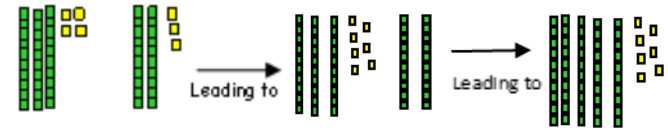


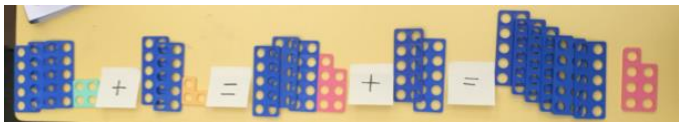
Numicon can be used in a similar style.

Partitioning (Augmentation model)

Base 10 equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.





Number line:



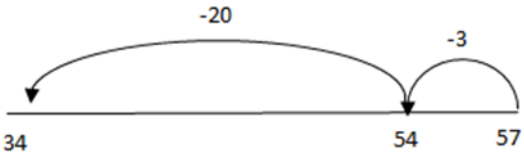
At this stage, children can begin to use an informal method to support, record and explain their method. (optional)

30 + 4 + 20 + 3

50 7

57

Number Line:



At this stage, children can begin to use an informal method to support, record and explain their method (optional)

(50 + 7) - (20 + 3)

30 4

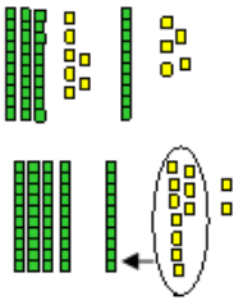
34

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

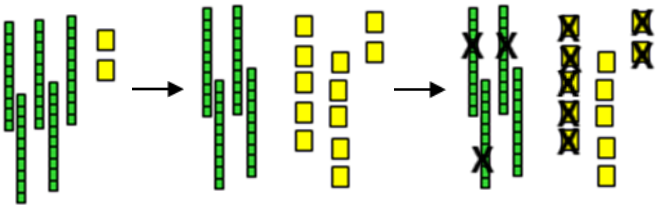
37 + 15 = 52



Discuss counting on from the larger number irrespective of the order of the calculation.

Base 10 equipment:

52 – 37 = 15

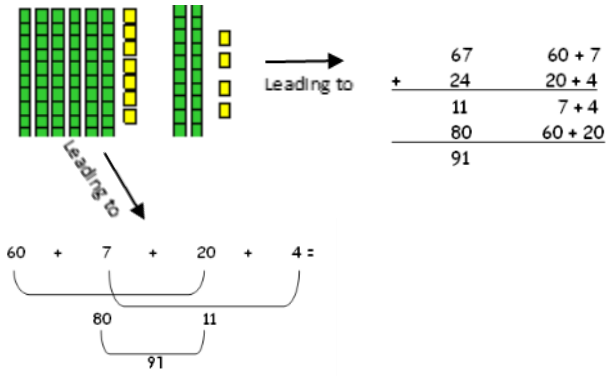


Expanded Vertical Method (optional)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.

Base 10 equipment:

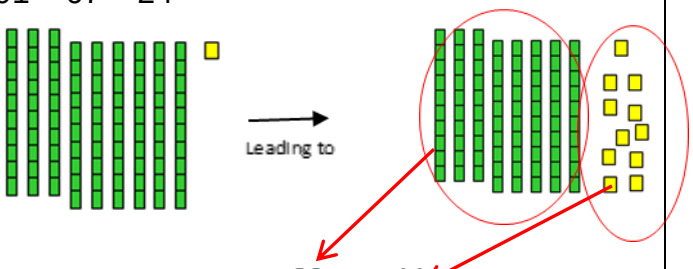
67 + 24 = 91



	67	60 + 7
+	24	20 + 4
	11	7 + 4
	80	60 + 20
	91	

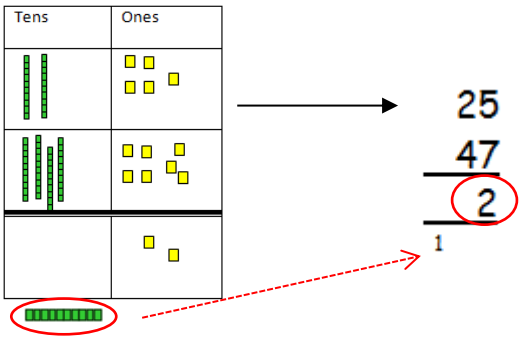
Base 10 equipment:

91 – 67 = 24



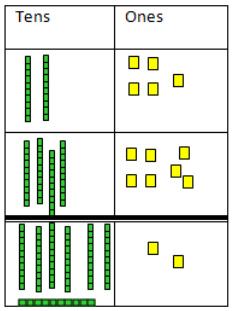
80 11
~~90~~ + ~~1~~
- 60 + 7
20 + 4

Compact method

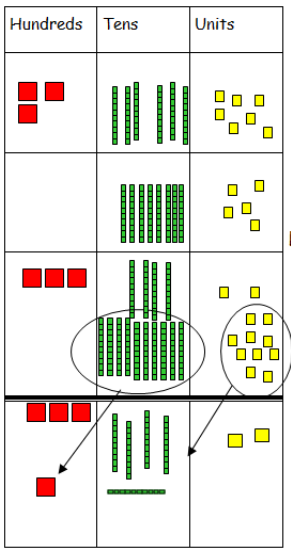


25
47
2
1

Leading to



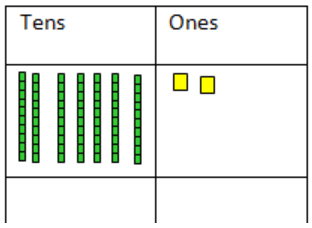
25
47
72
1



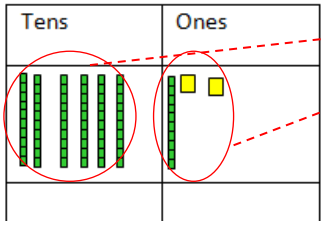
Leading to

3 6 7
+ 8 5
4 5 2
1 1

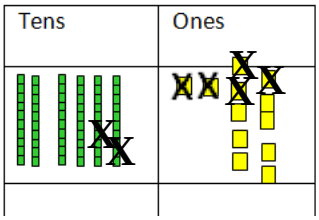
Compact decomposition



72
-25
47



6 1
~~7~~2
-25



6 1
~~7~~2
-25
47

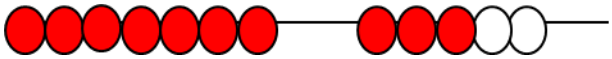
Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

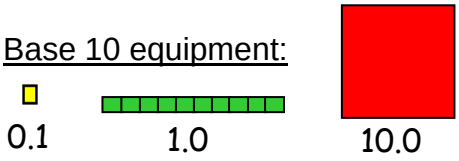
Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:

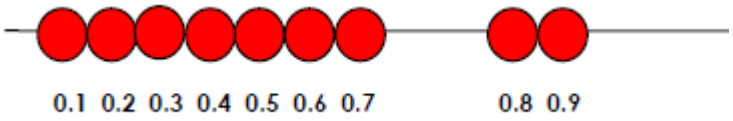


Addition of decimals

Aggregation model of addition

Counting both sets – starting at zero.

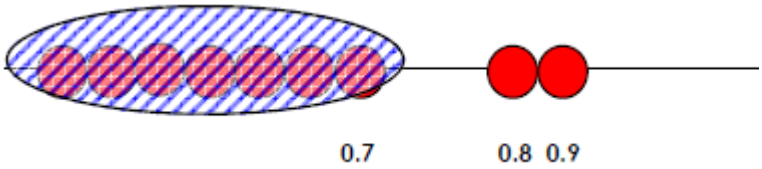
0.7 + 0.2 = 0.9



Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9



Bridging through 1.0

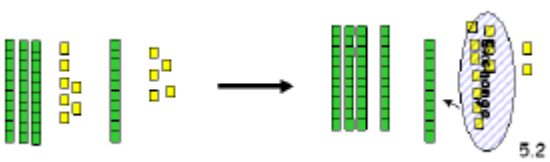
Encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning

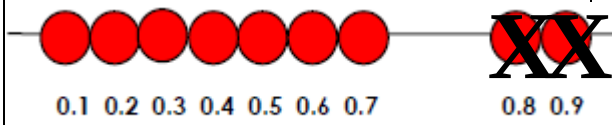
3.7 + 1.5 = 5.2



Subtraction of decimals

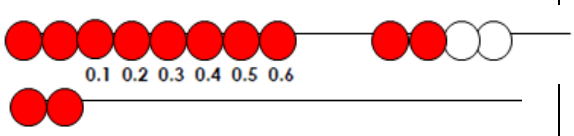
Take away model

0.9 – 0.2 = 0.7



Finding the difference (or comparison model):

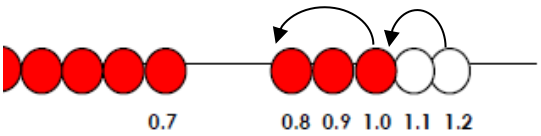
0.8 – 0.2 =



Bridging through 1.0

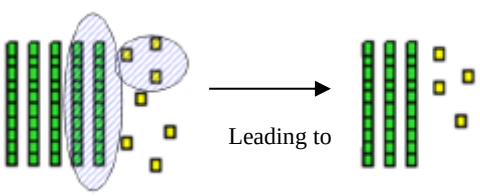
Encourage efficient partitioning.

1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

5.7 – 2.3 = 3.4



Gradation of difficulty- addition:	**Gradation of difficulty- subtraction:**
1. No exchange	1. No exchange
2. Extra digit in the answer	2. Fewer digits in the answer
3. Exchanging ones to tens	3. Exchanging tens for ones
4. Exchanging tens to hundreds	4. Exchanging hundreds for tens
5. Exchanging ones to tens and tens to hundreds	5. Exchanging hundreds to tens and tens to ones
6. More than two numbers in calculation	6. As 5 but with different number of digits
7. As 6 but with different number of digits	7. Decimals up to 2 decimal places (same number of decimal places)
8. Decimals up to 2 decimal places (same number of decimal places)	8. Subtract two or more decimals with a range of decimal places
9. Add two or more decimals with a range of decimal places	

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			

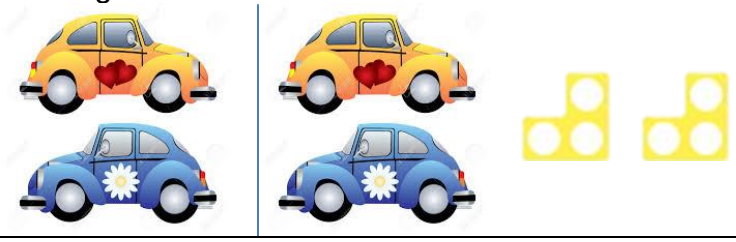


The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,
4 x 3 =12,
3 + 3 + 3 + 3 = 12,
4 + 4 + 4 =12.

And it is also a model for division

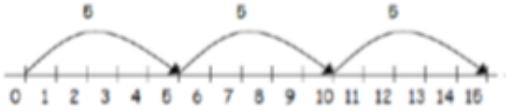
12 ÷ 4 = 3
12 ÷ 3 = 4
12 – 4 – 4 – 4 = 0
12 – 3 – 3 – 3 – 3 = 0

Multiplication	Division
Doubling Children to link two groups of the same size to doubling 	**Halving** Children to practice cutting thing (usually paper) in half and saying how many pieces there are. 
Early experiences Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem-solving activities involving equal sets or groups. 	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s. 

Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

Children learn that repeated addition can be shown on a number line.



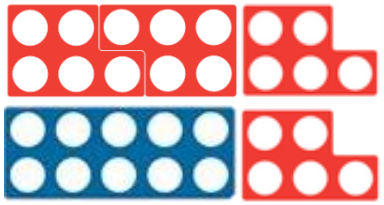
Children learn that repeated addition can be shown on a bead string.



Children also learn to partition totals into equal trains using Cuisenaire Rods

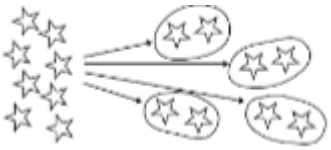


Children can use numicon to also show repeated addition and overlap it with tens as shown below.



Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.

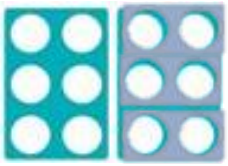


Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



Numicon can be used to show the total (left) and then twos to overlap (right).

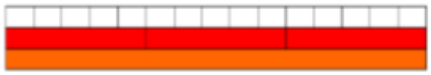


Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:

1 + 1 + 1 = 3 □ scaled up by 5 5 + 5 + 5 = 15



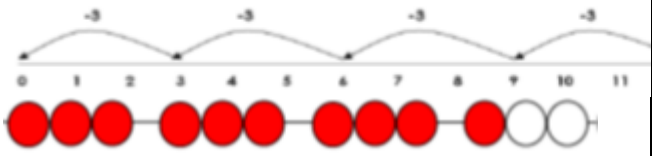
For example, find a ribbon that is 4 times as long as the blue ribbon.



We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.

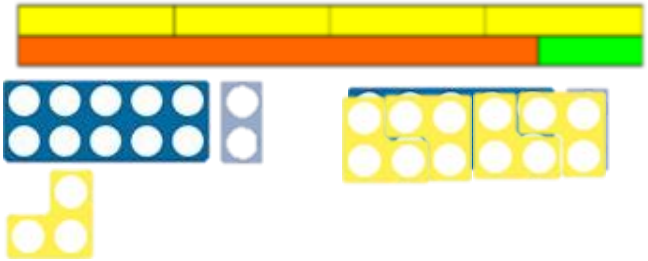
Repeated subtraction using a bead string or number line

12 ÷ 3 = 4



The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

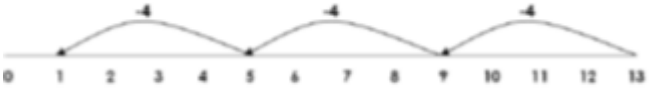
Cuisenaire Rods or Numicon also help children to interpret division calculations.



Grouping involving remainders

Children move onto calculations involving remainders. (bead strings and numicon can also be used effectively too)

13 ÷ 4 = 3 r1

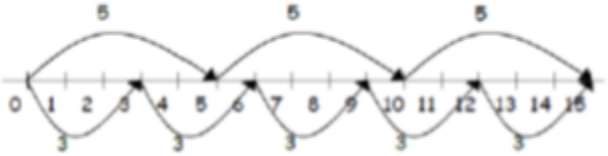


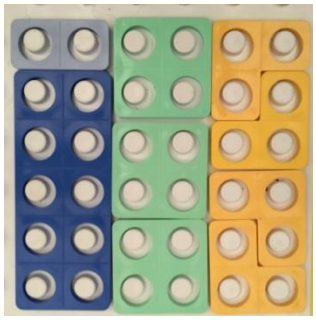
Commutatively

Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line.

3 x 5 = 15

5 x 3 = 15



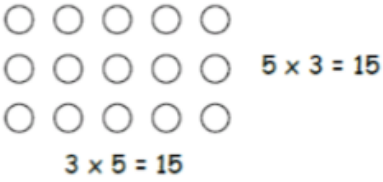


4 x 3 and 3 x 4 are the same.

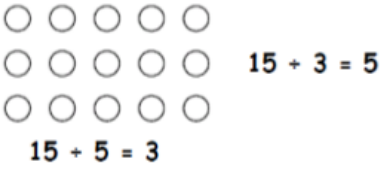
Children learn that division is **not** commutative and link this to subtraction.

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' /short division in a written method. This model also connects division to **finding fractions** of discrete quantities.



Inverse operations

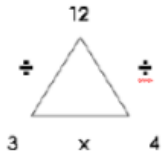
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3



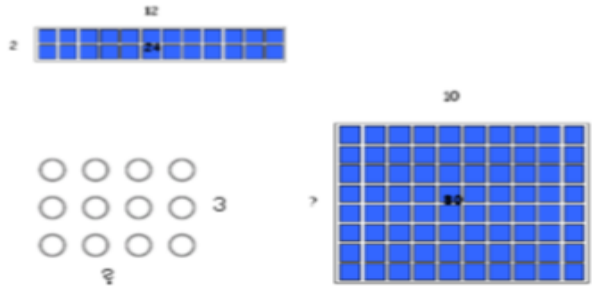
Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20 3 x Δ = 18 O x □ = 32

24 ÷ 2 = □ 15 ÷ O = 3 Δ ÷ 10 = 8

Click here for a link to a game that can be played using trios, cards and numicon.

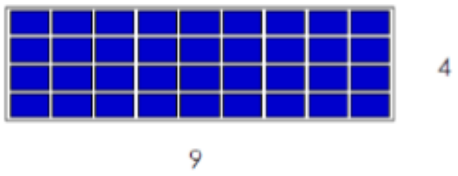
This can also be supported using arrays: e.g. 3 X ? = 12



Partitioning for multiplication

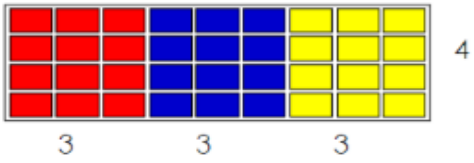
Arrays are also useful to help children visualise how to partition larger numbers into more useful representation.

9 x 4 = 36



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

9 x 4 =

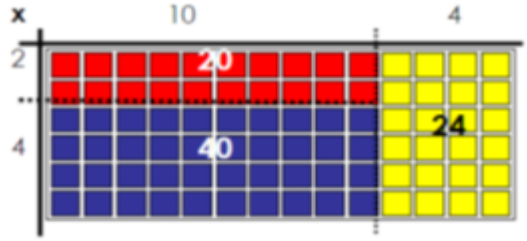


Which could also be seen as

9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36

Or 3 x (3 x 4) = 36

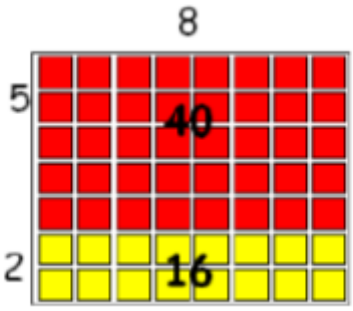
And so 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84



Partitioning for division

The array is also a flexible model for division of larger numbers

56 ÷ 8 = 7



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

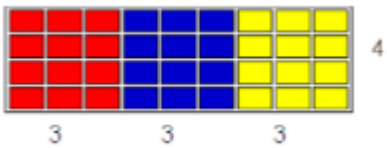
56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

To be successful in calculation, learners must have plenty of experiences of being flexible with partitioning. This is the basis of distributive and associative law.

Associative law (multiplication only):-

E.g. 3 x (3 x 4) = 36

The principle that if there are three numbers to multiply these can be multiplied in any order.

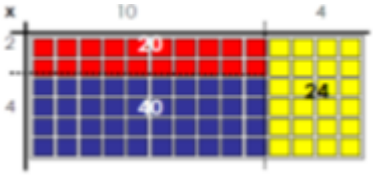


Distributive law (multiplication):-

E.g. 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84

This law allows you to distribute a multiplication across an addition or subtraction.

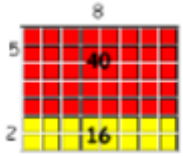
This could also be (6 x 10) + (6 x 4)



Distributive law (division):-

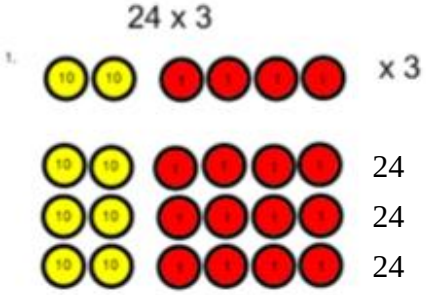
E.g. 56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

This law allows you to distribute a division across an addition or subtraction.



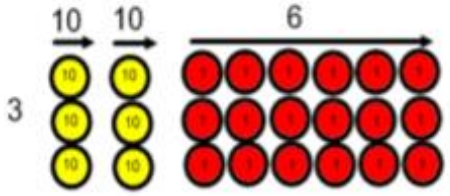
Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.
Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc.



Arrays leading into chunking and then long and short division

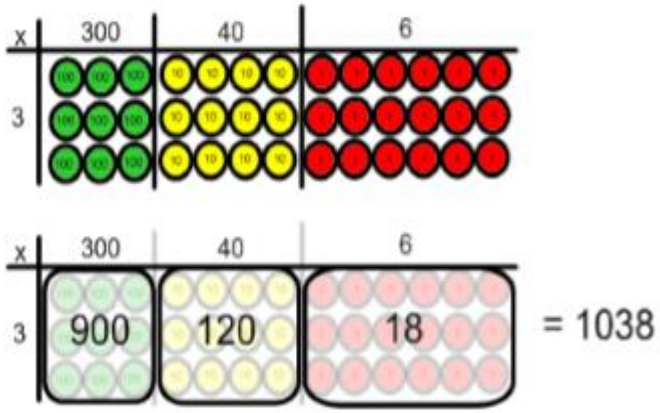
Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' and short method of division. Arrays are represented as 'grids' as a shorthand version.
e.g. 78 ÷ 3 =



78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =
10 + 10 + 6 = 26

Grid method

This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total.



The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters.
78 ÷ 3 =

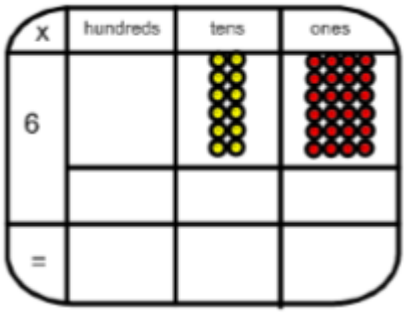
	78	
-	30	(10 x 3)
	48	
-	30	(10 x 3)
	18	
-	18	(6 x 3)
	0	

So 78 ÷ 3 = 10 + 10 + 6 =26

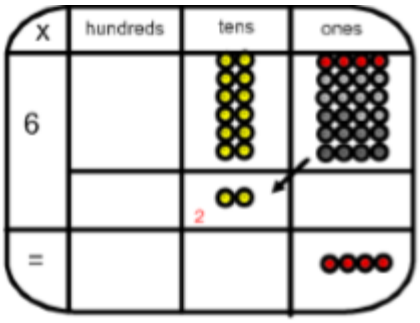
Short multiplication — multiplying by a single digit

The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging

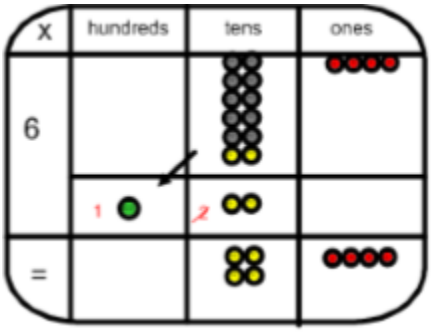
24 x 6



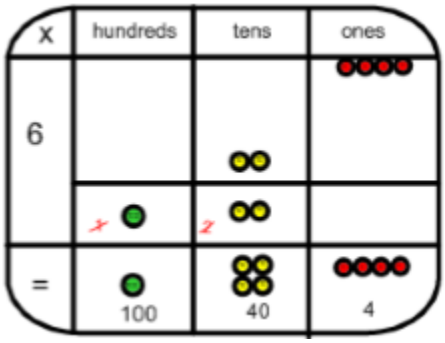
$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline \end{array}$$



$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 4 \\ \hline 2 \end{array}$$



$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 44 \\ \hline 1\ 2 \end{array}$$

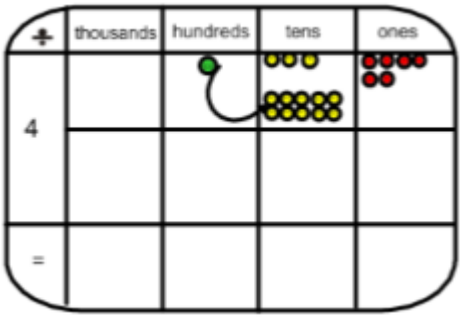


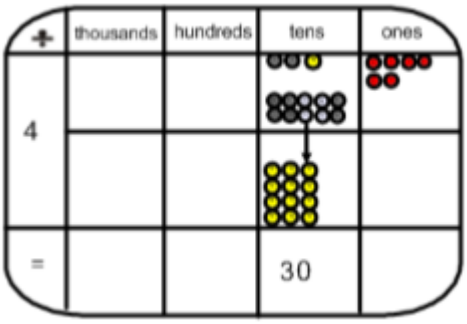
$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 144 \\ \hline \end{array}$$

Short division — dividing by a single digit

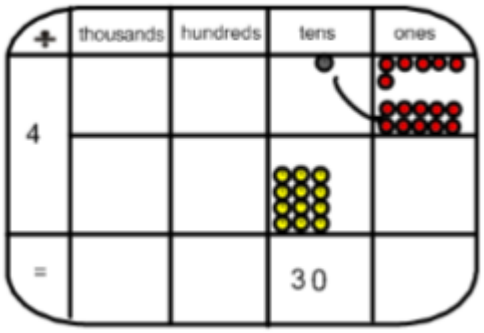
Whereas we can begin to group counters into an array to show short division working

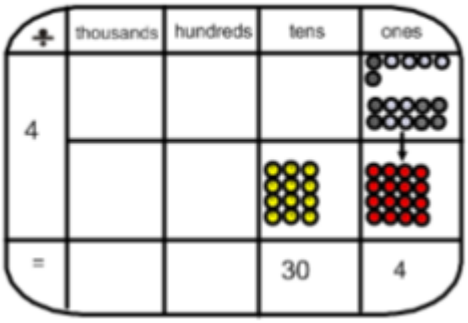
136 ÷ 4



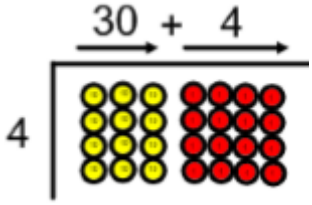


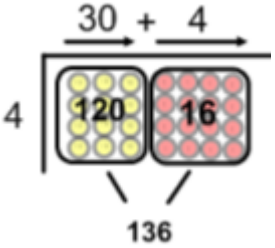
$$4\overline{)136}\quad 3$$





$$4\overline{)136}\quad 34$$



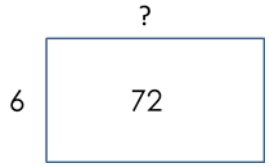


Gradation of difficulty (short multiplication)	**Gradation of difficulty (short division)**
1. TU x U no exchange 2. TU x U extra digit in the answer 3. TU x U with exchange of ones into tens 4. HTU x U no exchange 5. HTU x U with exchange of ones into tens 6. HTU x U with exchange of tens into hundreds 7. HTU x U with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x U 9. U.t x U no exchange 10. U.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O	1. TU ÷ U no exchange no remainder 2. TU ÷ U no exchange with remainder 3. TU ÷ U with exchange no remainder 4. TU ÷ U with exchange, with remainder 5. Zero in the quotient e.g. 816 ÷ 4 = **204** 6. As 1-5 HTU ÷ U 7. As 1-5 greater number of digits ÷ U 8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or U.12 ÷ 3 9. Where the divisor is a two digit number See below for gradation of difficulty with remainders
	Dealing with remainders Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g.: · I have 62p. How many 8p sweets can I buy? · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed? **Gradation of difficulty for expressing remainders** 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal

Long multiplication— multiplying by more than one digit Children will refer back to grid method by using place value counters or Base 10 equipment with no exchange and using synchronised modelling of written recording as a long multiplication model before moving to TOx TO and beyond.	Th H T U 5 9 X 2 6 3 5 4 (x6) 1 1 8 0 (x20) 1 5 3 4 1 1	**Long division —dividing by more than one digit** Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:- 1. Chunking model of long division using Base 10 equipment 2. Sharing model of long division using place value counters See the following pages for exemplification of these methods.

Chunking model of long division using Base 10 equipment

This model links strongly to the array representation; so for the calculation 72 ÷ 6 = ? - one side of the array is unknown and by arranging the Base 10 equipment to make the array we can discover this unknown. The written method should be written alongside the equipment so that children make links.

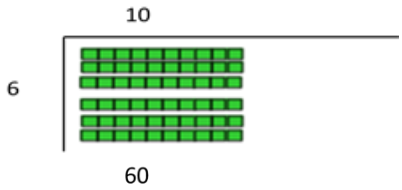


Begin with divisors that are between 5 and 9

72 ÷ 6 = 12



1. Make a rectangle where one side is 6 (the number dividing by) – grouping 6 tens



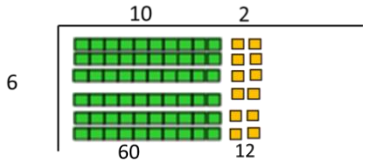
After grouping 6 lots of 10 (60) we have 12 left over



2. Exchange the remaining ten for ten ones
3.



4. Complete the rectangle by grouping the remaining ones into groups of 6



$$\begin{array}{r|l} 6 & 72 \end{array}$$

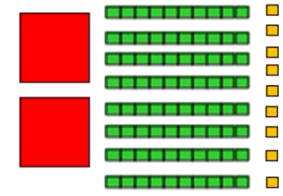
$$\begin{array}{r|l} & 1 \\ \hline 6 & 72 \\ -\,60 & (10\text{ x}) \\ \hline & 12 \end{array}$$

$$\begin{array}{r|l} & 1\,2 \\ \hline 6 & 72 \\ -\,60 & (10\text{ x}) \\ \hline & 12 \\ -\,12 & (2\text{ x}) \\ \hline & 0 \end{array}$$

Move onto working with divisors between 11 and 19

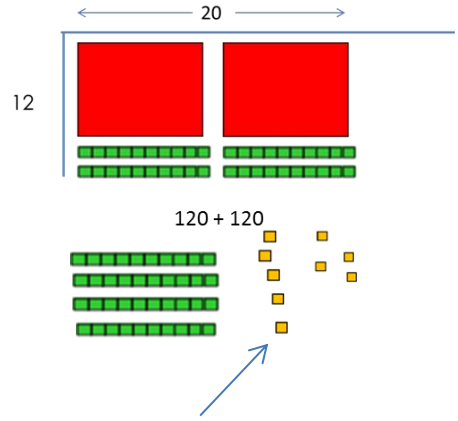
Children may benefit from practise to make multiples of tens using the hundreds and tens and tens and ones.

289 ÷ 12



$$\begin{array}{r|l} 12 & 289 \end{array}$$

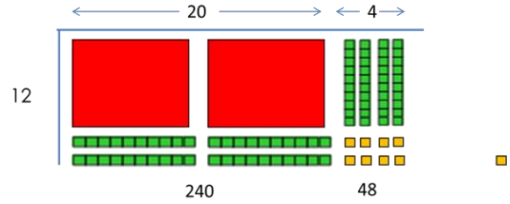
1. Make a rectangle where one side is 12 (the number dividing by) using hundreds and tens



With 49 remaining

$$\begin{array}{r|l} & 2 \\ \hline 12 & 289 \\ - & \underline{240} \quad (20 \times) \\ & 49 \end{array}$$

2. Make groups of 12 using tens and ones



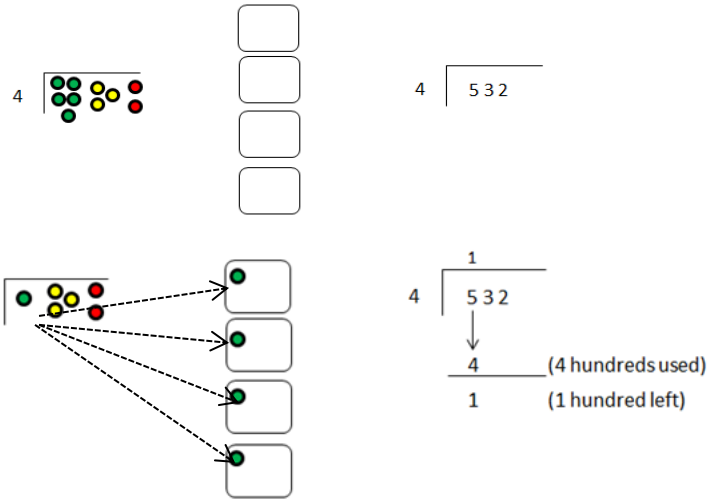
No more groups of 12 can be made and 1 remains

$$\begin{array}{r|l} & 24\ r1 \\ \hline 12 & 289 \\ - & \underline{240} \quad (20 \times) \\ & 49 \\ - & \underline{48} \quad (4 \times) \\ & 1 \end{array}$$

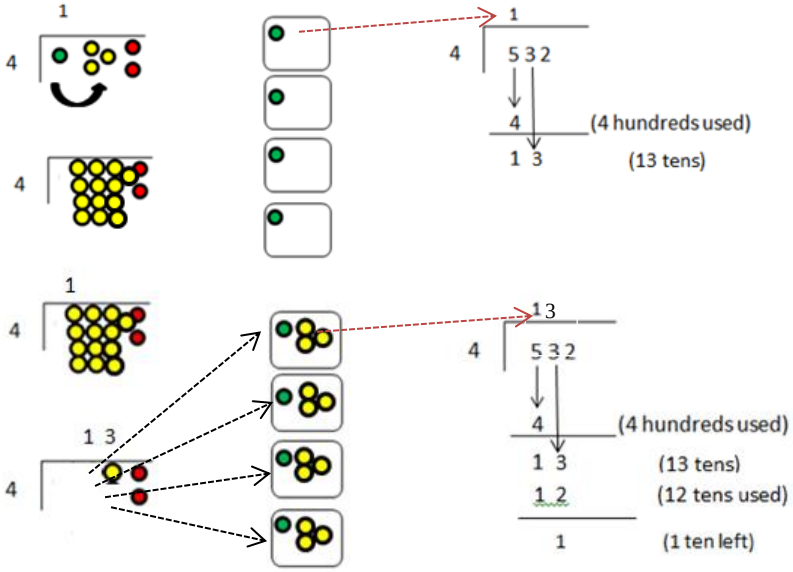
Sharing model of long division using place value counters

This is a development on from 'written calculation' part of this policy as it focuses on bringing 4 down not putting 400. Children must be able to do the method in the written calculation section first!

Starting with the most significant digit, share the hundreds. The writing in brackets is for verbal



Moving to tens – exchanging hundreds for tens means that we now have a total of 13 tens



Moving to ones, exchange tens to ones means that we now have a total of 12 ones counters (hence the arrow)

